STUDY OF FILTRATION FROM THE FRACTAL STRUCTURE IN POROUS MEDIA

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Hydrodynamic methods of studying porous media with a fractal structure are considered.

Following O'Shaughnessy and Procaccia [1], in this paper we consider the generalization of the equation of piezoelectric conduction to the case of filtration in porous media with a fractal structure. A real system with fractal properties has a characteristic length ξ (correlation length) such that on scales greater than ξ the system is homogeneous [2]. Roughly speaking, it can be represented as consisting of fractal blocks of size ξ. The problem of unsteady filtration in porous media with a fractal structure can be formulated upon satisfaction of the condition

$$\xi > l, \tag{1}$$

where ℓ is the characteristic scale of pressure gradient variation. Fractal structures for which Eq. (1) is satisfied will be called large-scale in contrast to the small-scale fractal structures of percolation theory, known as percolation clusters [3].

Fractals can form when one liquid expels another liquid from a porous medium. Large-scale fractal structures can arise when a bed (reservoir) is opened by the penetration of filtrates from drilling mud or cement slurry as well as when water, gas, and other agents are pumped in to maintain the reservoir pressure.

Nittman et al. [4] showed that diffusion-limited aggregation is a fairly general mechanism of fractal-structure formation. Fractal structures can thus be expected to form during colmatage of the bottom zone, the deposition of hydrocarbon solids as the oil temperature drops, and other such processes.

Let us consider radial filtration toward a borehole in a bed with a fractal structure. Suppose that $M(r, t)dt$ is the mass of fluid in an annular element of the bed of unit thickness, formed by cylindrical surfaces of radii r and $r + dr$:

$$M(r, t)\, dr = N(r)\, m(r, t)\, dr, \tag{2}$$

where $N(r)dr$ is the number of fractal nodes in an annular element:

$$N(r) = C_D r^{D-1}. \tag{3}$$

The law of conservation of mass of fluid can be written as

$$\frac{\partial M(r, t)}{\partial t} = -\frac{\partial G(r, t)}{\partial r}, \tag{4}$$

where $G(r, t)$ is the mass flow rate of fluid through a cylindrical surface of radius r.

The fluid and the pressure gradient are assumed to be related by

$$G(r, t) = \rho \frac{k(r)}{\mu} N(r) \frac{\partial P}{\partial r}. \tag{5}$$

It is natural to call $k(r)$ the conductivity of the fractal, referred to one of its nodes. Equation (4) should be considered as a relation determining $k(r)$, in much the same way as Darcy's law in the form $V = -(k/\mu)\partial P/\partial r$ determines the permeability k of a porous medium.

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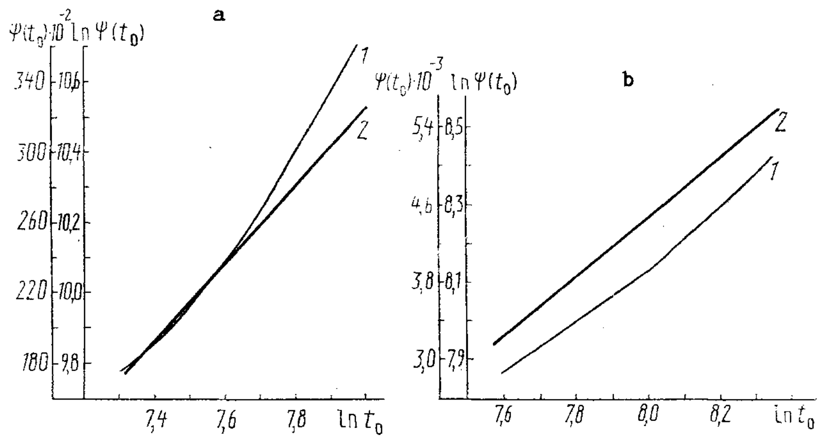

Fig. 1. Results from processing pressure build-up curve for wells No. 840 (a) and No. 151 (b) of the Taimurzinsk oil field in the coordinates: 1) $\psi(t_0)$-ln t_0 and 2) ln $\psi(t_0)$-ln t_0. $\psi(t_0)$, mPa·sec/m^3.

The conductivity of fractal structures obeys scaling laws. We assume, therefore, that

$$k(r) = k_1/r^\theta, \tag{6}$$

where θ is an exponent reflecting the anomaly of the conductivity, which originates from the very specific way in which the conducting nodes are combined into a fractal lattice [1]. When the fluid compressibility is taken into account we have

$$\frac{\partial m}{\partial t} = \rho V_0 \beta \frac{\partial P}{\partial t}. \tag{7}$$

From Eqs. (2)-(7) in the linear approximation we obtain the equation of piezoconductivity in a fractal

$$\frac{\partial P}{\partial t} = \frac{\varkappa}{r^\alpha} \frac{\partial}{\partial r}\left(r^\beta \frac{\partial P}{\partial r}\right), \tag{8}$$

where

$$\varkappa = K_1/\mu V_0 \beta_0, \quad \alpha = \mathrm{D} - 1, \quad \beta = \mathrm{D} - 1 - \theta = \alpha - \theta.$$

In the case of planar filtration $1 < \mathrm{D} < 2$, $0 < \alpha < 1$, and $\beta < 1$. Equation (8) resembles the equation of piezoconductivity in d-dimensional Euclidean space

$$\frac{\partial P}{\partial t} = \frac{\varkappa}{r^{\alpha-1}} \frac{\partial}{\partial r}\left(r^{d-1} \frac{\partial P}{\partial r}\right). \tag{9}$$

The quantities α and β in Eq. (8) may be fractional and differ ($\theta \neq 0$) from each other.

Equation (8) can be used to interpret the data from a transient study of a well. Let us consider, e.g., an operational method of processing pressure build-up curves (PBC's) [5, 6].

In the case of ordinary planar-radial filtration in a Euclidean space from Eq. (9) we can obtain ($d = 2$)

$$U(r, t_0) = AK_0(r/\sqrt{\varkappa t_0}),$$

where

$$U = \int_0^\infty P_1(r, t) \exp(-t/t_0)\, dt; \quad P_1 = P(r, t) - P_0(r);$$

$K_0(z)$ is a modified Bessel function. The constant A is determined from the boundary condition

$$-\frac{2\pi kh}{\mu} r_c \frac{\partial P_1(r_c, t)}{\partial r} = Q_0 - Q(t).$$

At $t > r_c^2/\varkappa$ we obtain

$$\psi(t_0) = \frac{U(r_c, t_0)}{F(t)} = \frac{\mu}{2\pi kh}\left(\ln\frac{1,26\varkappa}{r_c^2} + \ln t_0\right), \tag{10}$$

$$F(t_0) = \int_0^\infty (Q_0 - Q(t_0)) \exp(-t/t_0)\, dt.$$

In the case of ordinary planar-radial filtration, therefore, the PBC is rectified in the coordinates $(\Psi(t_0), \ln t_0)$ [5, 6].

Next we consider filtration in a fractal. Carrying out the Laplace transformation, we obtain for the function $U(r, t_0)$ the problem

$$\frac{1}{r^\alpha}\left(r^\beta U_r'\right)_r' - \frac{1}{\varkappa t_0} U = 0, \tag{11}$$

$$-\lambda r_c^\beta U_r'(r_c, t) = F(t_0), \tag{12}$$

where $\lambda = K_1 CD/\mu$.

For an infinite bed the limited solution of Eq. (1) has the form

$$U = BR^\nu K_\nu(R/\gamma\sqrt{\varkappa t_0}),$$

where $K_\nu(z)$ is a modified Bessel function of the ν-th order,

$$\nu = 1 - D/(2+\theta), \quad 0 < \nu < 1, \quad R = r^\gamma, \quad \gamma = \theta/2 + 1.$$

Determining B, from Eq. (12) we have

$$\psi(t_0) = \frac{R_c^{2\nu} K_\nu(z)}{\gamma\lambda[\nu K_\nu(z) + zK_\nu'(z)]}, \tag{13}$$

where

$$R_c = r_c^\gamma, \quad z = R_c/\gamma\sqrt{\varkappa t_0},$$

or, since $K_\nu' = -K_{\nu-1} - (\nu/z)K_\nu$, and $K_\nu = K_{-\nu}$,

$$\psi(t_0) = \frac{R_c^{2\nu} K_\nu(z)}{\gamma\lambda z K_{1-\nu}(z)}.$$

For small z we have

$$K_\nu \approx \frac{1}{2}\Gamma(\nu)\left(\frac{z}{2}\right)^{-2\nu}$$

and Eq. (13) becomes

$$\psi(t_0) = \frac{R_c^{2\nu}\Gamma(\nu)}{2\gamma\lambda\Gamma(1-\nu)}\left(\frac{z}{2}\right)^{-2\nu}$$

or $\ln\psi = a + \nu \ln t_0$, where

$$a = \ln\left[\frac{\mu\varkappa^\nu\Gamma(\nu)}{CK_1 D(\theta+2)^{1-2\nu}\Gamma(1-\nu)}\right]. \tag{14}$$

In the case of filtration in a fractal, therefore, the PBC is rectified in the coordinates $(\ln\psi, \ln t_0)$ and not in coordinates $(\psi, \ln t_0)$, as in the case of planar-radial filtration in Euclidean space. The value of ν can be estimated from the slope of the straight line (14).

As already mentioned, large-scale fractal structures can be formed when an oil field is flooded. Equation (8), therefore, can be used to study injection wells as well as development wells with a high degree of drowning, when the mobility of the oil during pressure build-up can be disregarded.

As an example, Fig. 1 shows the results obtained by processing PBC's in the coordinates ψ–ln t_0 and ln ψ–ln t_0. We see that the pressure build-up curves are rectified in ln ψ–ln t_0 coordinates. This phenomenon attests to the formation of fractal structures when the bottom zones of the wells are contaminated. Processing of the PBC's for 40 wells of the Bashkir oil fields confirmed that the state of bottom zones can be diagnosed for the porous medium with and without fractal properties.

NOTATION

$m(r, t)$, mass of fluid per fractal node; D, fractal dimension; C, K_1, proportionality factors; $G(r, t)$, mass flow rate of fluid through a cylindrical surface of radius r; ρ_1, μ, fluid density and viscosity; P, pressure; V, filtration velocity; k, permeability of the porous medium; $k(r)$, fractal conductivity per node; V_0, volume of fractal node; β_0, fluid compressibility coefficient; $P_0(r)$, initial steady-state pressure distribution; $Q(t)$, well production rate; Q_0, initial steady-state production rate; h, bed thickness; and r_c, well radius.

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